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TOWARDS A SECOND GRADIENT DAMAGE MODEL

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Abstract Global response of experimental uniaxial tests cannot be homogeneous because of the inevitable presence of localizations of deformation, that is always preferable from an energetic viewpoint. Accordingly one must introduce some characteristic lengths in order to penalize the too localized deformations. That leads to the concept of non-local damage models. The non-local approach implies non-local terms in the internal energy for controlling the size of the localization zone, see the remarkable works by Marigo and his coworkers [1, 2, 3, 4]. The introduction of a length scale in a fully non-local theory has been achieved in perydynamics, see e.g. [5, 6]. In this work the non-local term is given by the dependence of the internal energy of the second gradient of the displacement field. Following a variational approach [7], one defines the state variables and the total energy of the system associated with them. This total energy includes the potential energy (due to internal and external consequences) as well as the dissipated energy, here in terms of a damage parameter; finally, one formulates the evolution problem on the basis of the irreversibility of the damage parameter and of the variational principle. The irreversibility is taken because it is assumed that the damage field can not decrease its value. The variational principle assures that, at each time, a local minimum of the total energy among all accessible close states, is reached and the boundary conditions are derived and not assumed a priori.

Extended Abstract In this work the damage state of a material point x is charactrerized by a scalar internal variable $\alpha = \alpha(x)$, that is assumed to be within the range $\alpha \in [0,1]$, being $\alpha = 0$ corresponding to the undameged state and being $\alpha = 1$, on the other hand, corresponding to the broken material. At a given α the behaviour is elastic and defined by an internal energy $\mathfrak{U}$ that now is not only a functional of the displacement field $u(x)$ but also of the damage field $\alpha(x)$. If we interprete the damaged material as a microstructured material, we also let the internal energy to depend upon the second gradient of displacement in the following way,

$$\mathfrak{U}(u(x), \alpha(x)) = \int_0^L \left[\frac{1}{2}K(x)[\alpha(x)]^2 + \frac{1}{2}C(x,\alpha(x))\left[u'(x)\right]^2 + \frac{1}{2}P(\alpha(x))\left[u''(x)\right]^2\right] dx, \qquad (1)$$

where $K(x)$ is the resistence to damage that is assumed to be independent of damage, $C(x,\alpha(x))$ is the stiffness that is assumed to depend on damage in such a way that is zero for the broken material, i.e. $C(x,0) = 0$, $P(\alpha(x))$ is the second gradient parameter that gives the microstructural length scale (see [8, 9]).

Following the classic procedure of the variational approach we assume that the variation $\delta\mathfrak{U}$ of the internal energy $\mathfrak{U}$ is zero $\delta\mathfrak{U} = 0$ for every displacement variation δu, and for every damage variation $\delta\alpha > 0$

$$\delta\mathfrak{U} = 0, \qquad \forall \delta u, \delta\alpha > 0,$$

since the only admissible damage variation is positive, being impossible for the material to repair its state. The variation of $\mathfrak{U}$ must be computed

$$\delta\mathfrak{U} = -\int_0^L \left[\delta u \left(t - m'\right)' + \delta\alpha \left(K(x)\,\alpha(x) + \frac{1}{2}\frac{\partial C(x,\alpha)}{\partial\alpha}\left[u'(x)\right]^2 + \frac{1}{2}\frac{\partial P(\alpha)}{\partial\alpha}\left[u''(x)\right]^2\right)\right] dx + \left[\delta u\left(t - m'\right) + \delta u' m\right]_{x=0}^{x=L} \quad (2)$$

where the contact force t and biforce m are involved in the following form,

$$t = C(x, \alpha(x))\, u'(x), \qquad m = P(\alpha(x))\, u''(x) \quad (3)$$

The Euler-Lagrange equations are not only the classic ones with the related self-consistent boundary conditions,

$$\left(t - m'\right)' = 0, \qquad \forall x \in [0, L], \qquad \left[\delta u\left(t - m'\right) + \delta u' m\right]_{x=0}, \qquad \left[\delta u\left(t - m'\right) + \delta u' m\right]_{x=L} = 0. \quad (4)$$

but also the

$$K(x)\,\alpha(x) + \frac{1}{2}\frac{\partial C(x,\alpha)}{\partial\alpha}\left[u'(x)\right]^2 + \frac{1}{2}\frac{\partial P(\alpha)}{\partial\alpha}\left[u''(x)\right]^2 = 0 \quad (5)$$

Let us remark that the (5) is valid in the case of positive damage variation $\delta\alpha > 0$. In the case of negative damage variation $\delta\alpha < 0$, it is intended to set $\delta\alpha = 0$. Moreover, we note that the genaral damage Euler-Lagrange equation (5) is an algegraic equation in terms of the damage internal variable $\alpha(x)$. Thus, with a given and initial state of damage $\alpha(x)$, we solve eq. (4) with the insertion of eq. (3). Then we calculate the new displacement field $u(x)$ and, as a consequence, the new state of damage through eq. (5) for thos point x such that $\delta\alpha > 0$. We iterate such a procedure in order to show some numerical results. We will show that (i) an initial lack of stiffness concentrated in a given point x is enlarged in the numerical simulation and that (ii) the second gradient term (third addend in eq. (1)) makes such damage evolution slower.

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