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Active Strain Stabilized Fibrous Microstructures in Hyperelasticity

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Biological tissue is characterized by active processes of destruction and renewal at both the intercellular and intracellular scale. The mechanical structure of the material at both of these scales is typically characterized by highly complex morphological features, many of which can be characterized as fibrous in nature. Often it is these fibrous morphologies that confer to the material its bulk structural properties. Collagenous tissue is a notable example, although similar processes of destruction and renewal can occur in active composite materials that are not of biological origin. If one seeks to provide a continuum mechanics based procedure for analyzing any such material it is necessary to provide a reasonable accounting for the active processes of renewal and destruction. Examples of this for the case of collagenous materials can be found in [1], [2], [3] and [4].

Here we report on a general continuum mechanics based model that shows promise for describing such processes with respect to active fibrous microstructures under conditions of large strain. The particular processes of destruction and renewal for which this model applies is that in which all of the following *chemomechanical conditions* are met: (a) mechanical load bearing is hyperelastic in nature, (b) there is a fibrous component to the microstructure and the response of the fibrous component provides the dominant mechanical response compared to that of the non-fibrous component, (c) the change with time of the mechanical effect of the non-fibrous component is negligible (a special case of this is when the effect of the non-fibrous component is itself negligible), (d) the fibrous component process of *creation* proceeds at a constant rate, and (e) the fibrous component process of *destruction* does not proceed at a constant rate due to a dependence upon fiber stretching. While these five conditions may seem highly specialized, in fact they are motivated by the observed behavior of collagen fiber tissue as reported by Bhole et al. [5] and Robitaille et al. [6]. Of particular interest in that work is the observation that an increase in the collagen stretch serves to slow down the rate of destruction – a chemomechanical effect that couples fiber stretch to the efficacy of the enzymatic decay. Hence the microstructure in such tissues can be regarded as strain-stabilized relative to that of an unstrained material. On this basis, Hadi et al. [7] have recently supplied computational mechanics modeling and interpretation for such observations.

This work seeks to place such analysis methodologies within a broader hyperelastic framework so as to allow a systematic treatment of the complex structures and loadings that would characterize highly differentiated tissue assemblies or other heirarchical structures of such materials. The complete treatment of the overall framework is presented in [8]. This conference abstract presents an outline of the constitutive specification. It is also to be remarked that while we have provided continuum based treatments for actively changing fibrous morphologies (viz., [9]), the specific focus on aspects (d) and (e) in the above list of chemomechanical conditions provide the defining features that distinguish the present work from our earlier efforts. The material model requires the specification of five constitutive quantities:

- The free energy density for a macroscopically stable matrix constituent $W_m(I_1, I_2)$ where I_1 and I_2 are the first and second scalar invariants of the tensor $\mathbf{C} = \mathbf{F}\mathbf{F}^T$ and $\mathbf{F}$ is the deformation gradient tensor from the matrix natural configuration κ_o to the current configuration $\kappa = \kappa(t)$.

- The (constant) rate of fiber creation, $\chi_c > 0$.
- A means to determine the natural (stress-free) configuration of the fiber phase $\kappa_f(\tau)$ where τ is a time variable that gives the instant of fiber creation. The dependence on τ is due to the fact that fibers created at different instants could have different stress-free configurations. The rules for the determination of $\kappa_f(\tau)$ could make reference to the deformation gradient $\mathbf{F}(\tau)$. The determination of $\kappa_f(\tau)$ provides the linear mapping from $\kappa_o \to \kappa_f(\tau)$ and this linear mapping is denoted by $\mathbf{A}(\tau)$.
- The stored energy function $\psi(\mathbf{C},\mathbf{N},\mathbf{A})$ for a single fiber created at orientation $\mathbf{N}$ in κ_o. One such model involves taking any standard expression for a fiber Helmholtz free energy density $\Phi_f(I_4,I_5)$ from conventional anisotropic nonlinear elasticity, and then writing

  $$\psi(\mathbf{C},\mathbf{N},\mathbf{A}) = \Phi_f(I_4^{(\kappa_f\to\kappa)}, I_5^{(\kappa_f\to\kappa)}),$$

  where

  $$I_4^{(\kappa_f\to\kappa)} = \frac{\mathbf{N}\cdot\mathbf{CN}}{\mathbf{AN}\cdot\mathbf{AN}}, \qquad I_5^{(\kappa_f\to\kappa)} = \frac{\mathbf{A}^{-T}\mathbf{CN}\cdot\mathbf{A}^{-T}\mathbf{CN}}{\mathbf{AN}\cdot\mathbf{AN}}.$$

- A function $\hat{\eta}(\lambda_f) > 0$ for the rate of fiber dissolution where $\lambda_f = \sqrt{I_4^{(\kappa_f\to\kappa)}}$ is the fiber stretch.

On this basis the framework outlined in [8] provides expressions for stress, and also leads to a natural description of material homeostasis - namely a condition where the processes of creation and destruction are in complete balance for the prevailing conditions of material deformation.

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