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How Contact Interactions May Depend On The Shape Of Cauchy Cuts In N-Th Gradient Continua: Approach "À La D'alembert"

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The content of this presentation is mainly based on the paper [1] and on the Lecture Notes [2].
The Principle of Virtual Powers (or Virtual Works) is very ancient and has been always considered by those mechanicians who follow the ideas of D'Alembert and Lagrange as the basic concept in mechanics. More recently we can witness to a revival of the Principle of Virtual Powers: many authors finally recognize its fundamental role also in Continuum Mechanics. Actually the Principle of Virtual Powers (which can be regarded as a generalization of the Principle of Least Action, also when this last principle takes the form chosen by Hamilton and Rayleigh) has always been considered the most suitable conceptual basis for Continuum Mechanics: indeed this point of view, which maybe can be tracked up to Lagrange in his studies of fluid mechanics, has been used systematically in the famous works of E. and F. Cosserat and developed by Mindlin, Green, Rivlin, Toupin, Germain and many others. More detailed historical studies would be required to describe how and why the importance of the Principle of Virtual Works has been underestimated for long periods in the literature. In this paper it has been espoused the D'Alembertian view of mechanics as explicitly stated in the foreword of the Traité de Dynamique (1768) (see the paper cited at the beginning for an English translation of D'Alembert original words).

As clearly stated already in his works by Germain, the Principle of Virtual Works supplies the right tool for extending the Cauchy-Navier theory of Continuous Bodies. Actually the Cauchy-Navier format to Continuum Mechanics is not able to encompass the so-called Generalized or Micro-Structured Continua. These Continua are more and more attracting the interest of many researchers as they seem suitable to supply an efficient description of many phenomena, occurring in continuous systems in which at a "micro" lever are present some sharp variations of physical properties. Also many efforts are directed towards more or less mathematically rigorous homogenization procedures leading to continua which at a "macroscopic" level behave as higher gradient or Cosserat Continua.

More generally, higher order gradient theories are needed when "boundary layer" phenomena must be described: when considering impact phenomena

Here we want to show how the Principle of Virtual Powers, as formulated already by E. and F. Cosserat, allows us to generalize Cauchy representation formulas for contact interactions to the case of N-th gradient continua.
For this deduction it is crucial to consider the Principle of Virtual Powers valid for every subbody of considered continuous body, as done in all literature based on D'Alembert and Lagrange works.

We will also show that in N-th gradient continua contact interactions:

i) depend in a precise way on the shape of Cauchy Cuts; these cuts will be assumed to be constituted by a finite number of regular faces (orientable and for which all needed gradients of the unit surface normal field exist), a finite number of regular edges (i.e. curves on which are concurring two of the previously introduced faces and for which the Frénet basis and all its needed derivatives exist) and a finite number of wedges (i.e. points in which a finite number of edges are concurring).

ii) must include edge (i.e. concentrated on curves) and wedge (i.e. concentrated on points) interactions, so that one cannot assume that these interactions are represented in terms of surface integrals and

iii) cannot reduce simply to forces: indeed the concept of K-forces (generalizing similar concepts introduced by Rivlin, Mindlin, Green and Germain) is needed.

Indeed, in this paper we prove that "balance of force" is not a suitable postulate for developing the theory of N-th gradient continua.

References

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