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Wyle geometry and the nonlinear mechanics of distributed point defects

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In this seminar we show how to obtain the residual stress field of a nonlinear elastic solid with a spherically-symmetric distribution of point defects. The material manifold of a solid with distributed point defects - where the body is stress-free - is a flat Weyl manifold, i.e. a manifold with an affine connection that has non-metricity but both its torsion and curvature tensors vanish. Given a spherically-symmetric point defect distribution, we construct its Weyl material manifold using the method of Cartan's moving frames. Having the material manifold the anelasticity problem is transformed to a nonlinear elasticity problem; all one needs to calculate residual stresses is to find an embedding into the Euclidean ambient space. In the case of incompressible neo-Hookean solids we calculate the residual stress field. We then consider the example of a finite ball of radius R_o and a point defect distribution uniform in a ball of radius R_i and vanishing elsewhere. We show that the residual stress field inside the ball of radius R_i is uniform and hydrostatic. We then compare the nonlinear and classical linear solutions. We also prove a nonlinear analogue of Eshelby's celebrated inclusion problem for a spherical inclusion in an isotropic incompressible nonlinear solid.

The main idea of our geometric theory of defects has been presented in [1,2,3]. In this geometric framework one puts the defective body in a 3-manifold in which it is stress free. This is schematically shown in Fig. 1.

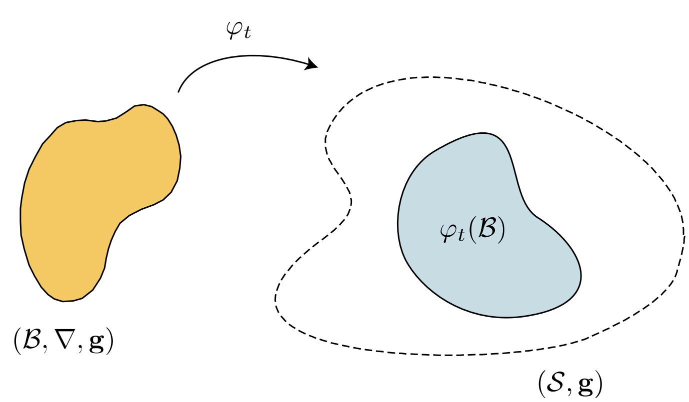

Figure 1. Comparison Kinematics of nonlinear anelasticity. Material manifold is a metric-affine manifold. Motion is a time-dependent mapping from the underlying Riemannian material manifold into the Riemannian ambient space manifold.

In the case of distributed point defects the relevant geometric object is non-metricity. Specifically, for a ball of radius R_o defect free outside a smaller sphere of radius R_i and with uniform point defect density n_0 we obtain the radial stress distribution and compare with the classical linear elasticity solution. This is shown in Fig. 2.

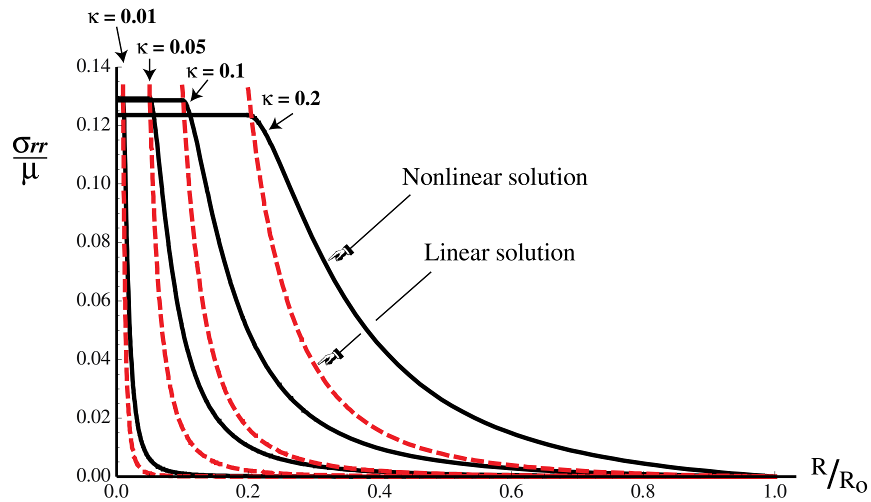

Figure 2. Comparison of the linear (dashed) and nonlinear (solid) solutions for the radial stress distribution for n_0=-0.1 and different values of $\kappa = R_i / R_o$.

Acknowledgments

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