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Fast and slow pressure waves electrically induced by non-linear coupling in Biot-type porous medium saturated by a Nematic Liquid Crystal

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Abstract Liquid Crystals are mostly known for their electro-optic properties. However, since a Liquid Crystal is usually assumed to be an incompressible fluid, very few researchers investigated the acousto-optic properties of these materials. However, some experiments conducted by Kim and Patel [1] show without any doubt that this interactions occur, and that further investigations on this phenomenon are needed. The non linear nature of Nematic Liquid Crystals (NLCs) and the presence of bifurcations make the analysis particularly challenging. In [2, 3] a non linear constitutive equation for pressure in nematic crystals is derived for describing this experimental evidence. In the present paper we consider a solid deformable porous matrix with interconnected pores, saturated with a nematic liquid crystal. From a mechanical point of view we assume that such a system can be described by means of a Biot-type model, limiting ourselves to the case of a solid matrix that has a negligible electric susceptibility and high porosity. We prove that an electrical stimulus applied on the NLC specimen may induce both type of Biot waves, fast and slow, along with shear waves in the porous matrix. This effect may be of use when one may wish to damp mechanically induced pressure waves using Darcy dissipation.

Description of the model Let us consider a domain $\mathscr{C}$ with boundary $\partial\mathscr{C}$, filled by a solid deformable porous matrix with interconnected pores, saturated with a Nematic Liquid Crystal (NLC) and let us denote with $\mathbf{r}$ a point inside $\mathscr{C}$. The starting point of our modeling procedure will the well established static theory of Nematics [4, 5], and the classic theory of poroelasticity [6], integrated with the results presented in [2, 3] for what concerns the coupling between the nematic director and the fluid density. In fact the NLC will be treated as a compressible fluid, with the addition of the constitutive equation which takes into account for the coupling among the distortion of the nematic field, the density of the liquid crystal and the deformation of the porous matrix. We will consider that electromagnetic-nematic evolution is quasi-static (still using standard Frank energy density), but when varying in time the applied external voltage we include mechanical inertial effects. These choice will allow us to observe numerically the initiation of coupled acoustic waves.
The state of the system is characterized by the following time varying spatial fields: the nematic director $\mathbf{n}(\mathbf{r},t)$, the electric potential $\phi(\mathbf{r},t)$, the displacement of the solid matrix $\mathbf{u}(\mathbf{r},t)$ and the pressure of the fluid $p_f(\mathbf{r},t)$.

The balance equations obtained from a variational principle are then

$$\kappa_{\mathsf{F}}\Delta\theta + \varepsilon_a\left(\nabla\phi\cdot\mathbf{n}(\theta)\right)\left(\nabla\phi\cdot *\mathbf{n}(\theta)\right) = 0, \quad \nabla\cdot(\mathbf{P}\nabla\phi) = 0 \tag{1}$$

$$\rho_{dr}\frac{\partial^2}{\partial t^2}\mathbf{u} - (\lambda + 2\mu)\nabla\nabla\cdot\mathbf{u} + \mu\nabla\times\nabla\times\mathbf{u} = (\varepsilon_p - \alpha_B)\nabla p_f \tag{2}$$

$$\Delta p_f - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}p_f = -\varepsilon_p\rho_f(\varepsilon_p - \alpha_B)\frac{\partial^2}{\partial t^2}(\nabla\cdot\mathbf{u}) - \frac{\rho_f}{\varepsilon_p}\frac{\partial^2}{\partial t^2}\omega_o \tag{3}$$

where $*\mathbf{n}$ denotes the Hodge star of $\mathbf{n}$, κ_{F} is the Frank Elastic coefficient, ε_a is the dielectric anisotropy, $\mathbf{P}$ is the permettivity tensor, ρ_{dr} and ρ_f are respectively the densities of the solid matrix and of the fluid, λ and μ are respectively the Lamé coefficients of the solid matrix, ε_p is the porosity, α_B is the Biot-Willis coefficient and c_0 is the speed of sound in the NLC. A very important role is played by $\omega_o = \delta|\nabla\cdot\mathbf{n}|^2/2 = \delta|\nabla\theta\cdot *\mathbf{n}|^2/2$, which is responsible for the coupling between the directors field and the density of the fluid and where δ is a coupling coefficient.
The associated boundary conditions are

$$\kappa_{\mathsf{F}}\nabla\theta\cdot\mathbf{m}_{\partial\mathscr{C}} = \mu_{\partial\mathscr{C}}, \quad (\mathbf{P}\nabla\phi)\cdot\mathbf{m}_{\partial\mathscr{C}} = (\mathbf{P}_{\mathrm{out}}\nabla\phi_{\mathrm{out}})\cdot\mathbf{m}_{\partial\mathscr{C}}, \quad \mathbf{u} = 0, \quad \nabla p_f\cdot\mathbf{m}_{\partial\mathscr{C}} = 0 \tag{4}$$

where $\mathbf{m}_{\partial\mathscr{C}}$ is the outer normal at the boundary, $\mu_{\partial\mathscr{C}}$ is the external torque, $\mathbf{P}_{\mathrm{out}}$ and ϕ_{out} are respectively the permittivity and the electric field on the outer side of the boundary of the NLC cell.

Numerical Results In all simulations we considered as reference configuration for the NLC an electrically unperturbed specimen with spatially constant pressure field. The response of the nematic liquid-crystal cell to a voltage input is computed using a commercial Finite Element Method software (COMSOL Multiphysics) by performing a time-dependent analysis with a BDF solver. The propagation of fast and slow waves is observed, as well as the presence of shear waves propagating in the solid matrix, Figure 1. The performed numerical simulations support the concept underlying the proposed model and further investigations, both theoretical and experimental, seem justified.

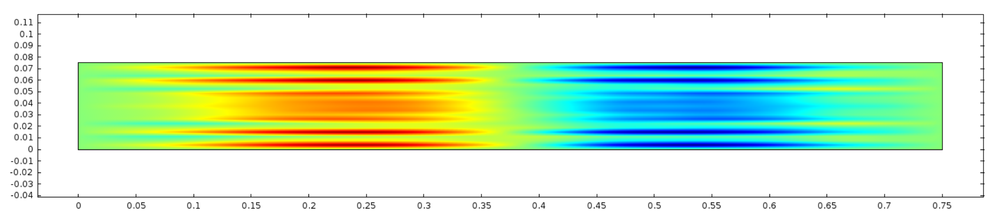

Figure 1. Fast shear waves propagating in the solid, matrix induced by an external electric field

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