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Large deformations in second gradient solids: some numerical simulations describing the onset of boundary layers

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In this paper are addressed three strongly related problems:

1) To find predictive models at micro- and meso-level for certain tissues which are used in high performance composite materials conceived in particular for aerospace engineering

2) To determine which class of models at macro-level (i.e. using continuum theories) is suitable to describe with sufficient approximation the behavior of aforementioned composite materials

3) To set up suitable micro-macro identification procedures for establishing the macro constitutive equations to be used in sep 2) starting from all micro informations obtained from step 1).

Moreover suitable numerical simulations are performed to prove how effective are the models formulated in points 1) and 2) when compared with experimental evidence. In particular, the conceived micro-macro identification procedure is used to supply the needed constitutive properties to the chosen macro-model.

There are many difficulties to be confronted: when using higher gradient continuum models (and in particular second gradient continua) it is clear that the standard integration schemes based on FEM could fail, because of the possible onset of (internal or boundary) interfacial layers. Indeed, to get numerically reliable integration schemes it is introduced a set of micromorphic kinematical descriptors and a suitable deformation energy which in the case of extremely stiff parameters (or by introducing suitable Lagrange multipliers and coupled constraints) reduce to second gradient models. This numerical recipe actually can find a physical counterpart in the actual microstructure of considered tissue: this point will merit subsequent investigations. In this context we refer to the results by Forest et al. (2011).

The introduction of second gradient terms in deformation energy regularizes the somehow strongly degenerate first gradient solution by allowing for the formation of boundary layers whenever a gradient of strain measure appears. The microstructures underlying the continuum models presented here tries to model some long range interactions among micro-structural elements which may show or may not show any preferred material direction, and therefore are isotropically or non-isotropically active. In this context the results found in dell'Isola et al. (2009) for isotropic second gradient continua may be of use, however for larger class of material symmetries the analysis found in Auffray et al. (2009) becomes necessary.

The numerical results obtained show that:

for a class of degenerated continuum models, which usually are introduced for tissues and which present -in the numerical integration schemes- great difficulties, suitable chosen second gradient models supply a regularization which seems to catch many physical phenomena, as the presence of internal boundary layers having non-negligible thickness and the quantitative behavior of strain energy close to the external boundaries of considered body.

For a large class of micro- and meso-models it is proven the possibility of describing effectively the short-range and long-range interaction phenomena occurring in the considered micro- and meso- structures.

For a large class of micro- and meso-model it is proven that a simplified continuum model can be introduced which is strongly non-linear in the first gradient part and may be approximately considered to be linear in the second gradient terms.

The presentation is concluded by listing the greater number of problems which are opened by the results which were already obtained:

In the considered strongly non-linear first gradient models, the concentration of stress in corners, which is a weak point of linear first gradient theories and which is cured by regularizing the boundaries or by suitably regularizing the model, does not seem to be easily removable with the standard procedures. One has to clarify if this is a not-well-understood physical feature of considered tissues or if this is a numerical or mathematical weakness of considered models

The kind of degeneracy of considered first gradient models does not seem to have been studied before so carefully as it deserves. Therefore mathematical methods need to be developed in order to understand why second gradient models cure the know degeneracies, eventually proving suitable existence (and/or uniqueness) theorems

The class of meso-models which are producing, after homogenization, the considered degenerate models needs to be characterized. Moreover theorems of convergence of micro- and meso-models need to be proven in order to understand in which sense macro-models can approximate considered micro- and meso-models. As a byproduct of these results one hopes to be able to identify -based on more rigorous procedures- the macro parameters in terms of micro parameters. In this context the micro-long-range interactions may play a relevant role.

References

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