

ANALYTICAL MECHANICS OF CONTINUA WITH MICROSTRUCTURE

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The most general and elegant axiomatic framework on which continuum mechanics can be based starts from the Principle of Virtual Works. This Principle, which was most likely used already at the very beginning of the development of mechanics became after D'Alembert the main tool for an efficient formulation of physical theories. Also in continuum mechanics it has been adopted soon. Indeed the Principle of Virtual Works becomes applicable in continuum mechanics once one recognizes that to estimate the work expended on regular virtual displacement fields of a continuous body one needs a distribution (in the sense of Schwartz). Indeed in the present paper we prove, also by using concepts from differential geometry of embedded Riemannian manifolds, that the Representation Theorem for Distributions allows for an effective characterization of the contact actions which may arise in N -th order strain-gradient multipolar continua (as defined by Green and Rivlin in 1964), by univocally distinguishing them in actions (forces and n -th order forces) concentrated on contact surfaces, lines (edges) and points (wedges). The used approach reconsiders the results found in the pioneering papers by Green and Rivlin (1964)-(1965), Toupin (1962), Mindlin (1964)-(1965) and Casal (1961) as systematized, for second gradient models, by Paul Germain (1973). Finally we indicate how Euler-Cauchy approach to contact actions and the celebrated tetrahedron argument may be adapted to N -th order strain-gradient multipolar continua.