



*International Research Center on*  
**MATHEMATICS AND MECHANICS**  
**OF COMPLEX SYSTEMS**

**ABSTRACT**

**Metamaterial design: a challenge in mathematical physics and engineering Sciences**

The novel demands from the science of materials require the conception of materials showing exotic behaviour.

We claim that an old problem from the theory of analog circuits must be revived. Indeed before the invention of digital computers in engineering and scientific applications during the decades 1930-1960 analog computers were used. Given a PDE the experts of analog circuits could design a specific circuit governed by the given equation. When successful they had obtained the circuit which was the analog of the physical system which was modelled by their PDE.

In our more modern setting we specify the mechanical properties of a given material or structure by assigning the action functional which governs it and then we look for the specific mechanical system whose behaviour is exactly the desired one.

We talk about the <<synthesis>> of a specific architected material or of a specific structure.

In some situations homogenisation techniques are needed and the required material is synthesised at a certain length scale by using some smaller scale microstructures.

In this context the theory of higher gradient continua needs to be introduced and it presents some mathematical problems whose solution needs the methods of differential geometry and mathematical physics. The concepts of stress and strain must be generalised to encompass this more general case, which cannot be framed in the standard theory of continua by Cauchy, as formalised by Truesdell.

In other situations the synthesis problem can be solved only by looking for the best value for some parameters characterising the considered microstructure.

In this context the theory of optimisation plays a crucial role.

In this presentation we refer some results recently obtained in the problem of synthesis

of second gradient materials, of extremely extensible microstructures and of structures being damage resistant.

The synthesis problem is solved by introducing so called <<pantographic>> structures. Their mathematical properties are discussed, some numerical simulations showing their exotic behaviour are presented and some experimental evidence is shown,